O.R. Applications

Investment optimization on port’s development by fuzzy integer programming

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Abstract
In this research, the main purpose is to formulate a model to determine the optimum investment on port development from national investment prospective; on the other hand, costs and benefits are calculated from consumer and investor’s viewpoint. The formulated model is an integer-programming model. The emphasis is on how to formulate an investment optimization problem where cargo operation, investment costs, cargo-handling capacity, cargo transportation network, and the world maritime fleet constraints are included. Fuzzy numbers are used for cargo forecast study results. The output of the model is the type of design ships and design berths which are needed in each sub period, so that the port planner (the government) will find out the optimum development plan of port in each sub period when there is uncertainty in cargo handling forecast (fuzzy numbers).

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1. Introduction

The development of transportation infrastructures has had a great influence on financial and political situation inside countries. In countries like Iran, where more than 90% of cargo transportation (export, import and transit) are done via maritime boundaries, port development is important. Therefore, emphasis should be put on investment in these areas to expand their capacity. Unfortunately, budget limitations restrict the dynamic plans of these countries. There has not been any mathematical method to find the optimum solution for port development. In this paper, the purpose is to formulate an investment model to find the optimum investment steps by application of operational research science and fuzzy logic concept to model the available uncertainties. So fuzzy integer linear programming models are used to determine the optimum investment and...
development of a port. In order to formulate this model, Shahid Rajaee port is chosen as a case study port, which has the largest container terminal among southern ports of Iran. The government must develop Shahid Rajaee port to increase its capacity because the container transportation is going to be the most common mode of maritime transportation. On the other hand, seagoing international trade moved by container ships is a major factor in most of the countries. The current cargo handling capacity of the port is about 1,450,000 TEU\(^2\) per year. The cargo forecast results claim that the government should increase its capacity up to about 12,000,000 TEU per year by the year 2030 (Halcrow, 2004). Considering the world maritime fleet development different scenarios should be built and a suitable model will be developed. The most important decision criterion for this project is the budget. Finding an optimum solution, which can lead the investor to develop ports with the lowest costs, has a great importance for developers.

There are different approaches to solve a fuzzy linear program such as Zimmermann approach (Zimmermann, 1996), Chanas approach (Chanas, 1983) and Julien approach (Julien, 1994). According to the results of the cement transportation studies in Taiwan (Shih, 1999), which is a mixed integer programming, the results of these three approaches are close to each other. It is noticeable that Herrera and Verdegay (1995) solved three types of fuzzy integer linear programming using fuzzy constraints, imprecise costs, and fuzzy coefficients. The other solution for solving integer programming is the Chanas solution for integer transportation problem (Chanas and Kuchta, 1998). In this study because of the large scale of this problem the Julien’s approach is used to yield the optimum result. If the problem was continuous optimization it would be possible to solve the problem by using Shian-Tai method too (2004). The problem in this study is a complicated and developed form of fixed-charge problem (Hillier and Lieberman, 2001) with fuzzy numbers on the right hand side of some constraints.

2. Preliminaries

In this section a brief definition of fuzzy numbers, \(\alpha\)-cut and Julien’s approach are given.

2.1. Fuzzy numbers

A fuzzy number \(\tilde{M}\) is a convex normalized fuzzy set \(\tilde{M}\) of the real line \(R\) such that:

I. It exists exactly one \(x_0 \in R\) with \(\mu_{\tilde{M}}(x_0) = 1\).

II. \(\mu_{\tilde{M}}(x)\) is piecewise continuous (Zimmermann, 1996).

In this research, all fuzzy numbers are shown with \(\sim\) symbol.

2.2. \(\alpha\)-cut

The crisp set of elements that belong to the fuzzy set \(\tilde{A}\) at least to the degree \(\alpha\) is called the \(\alpha\)-cut, (Zimmermann, 1996):

\[A_\alpha = \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\} \]

2.3. Julien’s approach for solving fuzzy LP models

In this research, Julien’s approach will be used to solve the integer linear problem of the model. So the approach is briefly described. Julien (1994) proposed an approach that incorporates the \(\alpha\)-cut concept with Buckley’s (1989) possibility programming to resolve the problem including fuzzy objective and fuzzy RHS\(^3\).

\(^2\) Twenty Equivalent Unit.
\(^3\) Right Hand Side.
by solving pairs of crisp linear programming problems in expressions (2) and (3). Expression (1) is a general form of fuzzy LP model.

\[
\text{MAX } \bar{c}x \\
s.t. \quad a_i \bar{x} \leq b_i, \quad i = 1, \ldots, m, \\
\text{MAX } c^\alpha_L x \\
s.t. \quad A^\alpha_U \leq b^\alpha_L, \quad x \geq 0, \\
\text{MAX } c^\alpha_U x \\
s.t. \quad A^\alpha_L \leq b^\alpha_U, \quad x \geq 0.
\] (1) (2) (3)

The superscript represents an \( \alpha \)-cut of the fuzzy parameters, and the subscripts L and U denotes the corresponding lower and upper cuts. The triangular possibility distribution function is often assumed for imprecise problem.

3. Fuzzy LP model for investment problem on port

3.1. Methodology of the research

According to the results of cargo forecast studies, the needed cargo handling capacity is estimated. Different scenarios can be considered for port development but the optimum scenario from the national viewpoint is chosen. In accordance with the unknown factors, which may influence the cargo forecast studies, there is an uncertainty for cargo forecast that is why fuzzy numbers are used. It means that for a significant uncertainty and by considering all possible scenarios, the optimum development is determined. Therefore, the steps of the study are

1. Gathering necessary data for model.
2. Determining the cargo handling capacity for port by the end of each study sub period.
3. Creating the objective function from national viewpoint.
4. Making the constraints.
5. Solving the optimum model.
6. Analyzing the results of the optimum answer.

3.2. Needed data

1. It is necessary to know who is going to invest in improving the ports. Ports can be developed by three types of investor: private sector, government, and BOT.\(^4\) The costs and benefits differ from one viewpoint to another. In this study, the emphasis is on how to optimize the port development plan from the national viewpoint.
2. Ship’s generation and their traveling distance on different routes must be taken into consideration so that the port is built cost effective on the kind of ships that are berthed there. Large ships have some constraints in their traveling routes and number. So the cost effectiveness in building berths that accommodate large ships must be analyzed.
3. Cargo forecast study results in each sub period are necessary.
4. Cargo distribution must be determined by imported containers and exported ones.

\(^4\) Built Operation and Transfer.
5. If there are any physical constraints, they must be considered. Such as geographical constraints and geological constraints.

6. The importance of weighting up costs and benefits related to consumers and investors must be clear to the analyzer.

3.3. Assumptions

I. Devaluation of currency considered to be 15% for Iran.

II. Useful lifetime for berths and CSD\textsuperscript{5} dredging operation considered to be 50 years. The standard lifetime for a gantry is about 25 years but case study port’s climate, reduces that lifetime to around 15 years.

III. The study period divided into sub periods of 5-years.

IV. Five types of container ships according to Iran’s maritime policy and growing world trends are candidates. Their capacities are 4000, 6000, 8000, 10,000 and 12,000 TEU. Their properties are given in Table 1 (Lloyds, 2000).

V. Berth occupancy assumed to be 0.55. UNCTAD (1985)\textsuperscript{6} proposes berth occupancy to be more than 0.55 but according to the delay time importance for large container ships, nowadays port planners avoid facing any delays in ports. The results of some studies which are carried out in different ports of the Persian Gulf show that if the berth occupancy assumed to be 0.55 then the waiting time will decrease to zero. So in this study port occupancy is assumed to be 0.55 (Halcrow, 2004).

VI. In accordance with cargo forecast results; triangular fuzzy numbers are used for cargo handling in Shahid Rajaee port.

VII. The importance of weighting up costs from government and consumer viewpoints assumed to be equal.

3.4. Objective function

In this research, the objective function is to minimize the net present value of costs from national viewpoint. The costs and benefits related to consumers and investors must be considered. The fuzzy numbers of containers that must be handled are given by cargo forecast studies. The objective function is

$$ G = \text{MIN}(C_{\text{transportation}} + C_{\text{facility}} + C_{\text{dredge}} + C_{\text{operation}} - B). $$

The objective function includes the net present value (NPV) of transportation costs, facility costs, dredging cost, operation costs, and benefits gained from foreign shipping lines traveling to the berth.

Where,

$$ C_{\text{transportation}} = \sum_n \sum_i \sum_m c_{imn} x_{imn} \times f(P/F, r, n) \times f(P/A, r, t). $$

This sentence calculates the net present value of transportation costs.

\textit{i}: Denotes the index of ship’s type. The capacity of the candidate ships assumed to be equivalent by their index, \((i \in I, I = \{1, 2, 3, 4, 5\})\). For example \(i = 1\) is the ship type with capacity of 4000 TEU, \(i = 2\) is the ship type with capacity of 6000 TEU, \(\ldots\)

\textit{n}: Denotes the start year of each interval \(t\), \((n \in N, N = \{n_0, n_1, n_2, \ldots, n_N\})\).

\textit{t}: In this study the total period of analysis is divided into some equal sub periods \(t\). Obviously, the port capacity must be greater than the cargo forecast by the end of each interval, \(t = n_1 - n_0 = n_2 - n_1 = \ldots = n_N - n_{N-1}\).

\textit{m}: Denotes the maritime route, \((m \in M, M = \{1, 2, \ldots, m\})\).

\(x_{imn}\): Integer variable used to model the number of the ships type \(i\) traveling in year \(n\) and by route \(m\).

\textit{r}: Devaluation of currency.

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\textsuperscript{5} Cutter Suction Dredge.

\textsuperscript{6} United Nations Conference on Trade and Development.
The coefficient used to convert the future cost in year \( n \) to the present value. In this formula \( P \) denotes the present value and \( F \) denotes the future value.

\[
f(P/F, r, n) = \frac{1}{(1 + r)^n}, \quad P = \frac{F}{(1 + r)^n}.
\]

The coefficient used to convert the annual cost in sub period \( t \) to the present value. \( A \) denotes the annual value.

\[
f(P/A, r, t) = \frac{(1 + r)^t - 1}{r(1 + r)^t}, \quad P = A \times \frac{(1 + r)^t - 1}{r(1 + r)^t}.
\]

c\(_{inn}\): Transportation cost when the cargo is traveling by ship \( i \) in year \( n \) and by route \( m \). This cost is a variable cost in the model and its formula is

\[
c_{inn} = T_i \times l_m \times c_{um}.
\]

\( T_i \): The capacity of ship’s type \( i \).

\( l_m \): The travel distance of route \( m \).

\( c_{um} \): The transportation cost per TEU per nautical mile in year \( n \).

In order to clarify the meaning of \( n, t \) they are illustrated in Fig. 1.

\[
C_{\text{facility}} = \sum_k \left( \sum_i b'_{ik} \times q_{ik} \times f(P/F, r, v(k)) + \sum_i \left( \sum_{k'=1}^{k'} b''_{ik} \times (1 + f(P/F, r, 15)) + \sum_{k=k'+1}^{K} b''_{ik} \right) \right) \times q_{ik} \times f(P/F, r, v(k)).
\]

\( k \): Number of intervals \((k \in K, K = \{1, 2, \ldots, k\})\).

\( k' \): Denotes the number of the last time interval that the investor must substitute the gantries. \( k' \) will be determined by the length of sub periods, the practical lifetime of gantries and total study period. In this research, the length of sub periods assumed to be 5 years, the practical lifetime of gantries assumed to be 15 years and the total study period is 25. So \( k \) will be 5 and \( k' \) will be 3, \((t = 5, n = \{0, 5, 10, 15, 20, 25\}, k = 5, k' = 3)\). It means that if the investor has to substitute gantries, which their expectancy life have been expired during study period.

\( v(k) \): The year that interval \( k \) starts, \((v(k) \in V(k), V(k) = \{0, 5, 10, 15, 20\})\).

\( b'_{ik} \): Capital investment needed for berth type \( i \) construction in interval \( k \). The construction of berths that are needed in interval \( k \) must be started from the beginning of the interval, \( v(k) \).

\( b''_{ik} \): Capital cost needed to buy gantry crane \( i \) in interval \( k \).

\( b''_{ik} = (\text{gantry price})_{ik} \times (\text{gantry number}) \).

\[
\begin{array}{cccccccccc}
\text{Ship capacity (TEU)} & & & & & & & & & \\
4000 & 267 & 40 & 11.8 & & & & & & \\
6000 & 290 & 43 & 12.5 & & & & & & \\
8000 & 340 & 43 & 13.6 & & & & & & \\
10,000 & 350 & 50 & 14.0 & & & & & & \\
12,000 & 380 & 53 & 14.6 & & & & & & \\
\end{array}
\]
$q_{ik}$: Integer variable used to model the number of berths, which are needed to be built in interval $k$.

$f(P/F, r, 15)$: The coefficient used to calculate the net present value of gantry cranes that must be prepared at year $v(k) + 15$.

$f(P/F, r, v(k))$: The coefficient used to calculate the net present value of expenditures in year $v(k)$.

$$C_{\text{dredge}} = \sum_n \text{dredge}_n^{'} \times f(P/F, r, n).$$  \hspace{1cm} (7)

There are two kinds of dredging: Initial dredging which must be done whenever a greater ship is going to berth in port and annual dredging which removes annual sediments. For example, about 500,000 m$^3$ dredging is done in Shahid Rajaee port each year.

This sentence calculates the net present value of initial dredging costs during the whole study period.

$dredge_n^{'}$: Capital dredging cost in year $n$, this cost must be paid when the design ship in the current sub period needs greater draft than available depth. $dredge_n^{'}$ will be calculated by using the model. Eqs. (12) and (13) represent how to find it.

$$C_{\text{operation}} = \sum_i \sum_k (l' b_i^j + l'' b_i^j) \times f(P/A, r, nN - v(k)) \times f(P/F, r, v(k)) q_{ik} + \sum_n \text{dredge} \times f(P/A, r, nN).$$  \hspace{1cm} (8)

In this formulation, the net present value of total maintenance and annual dredging cost is calculated.

$l' b_i^j$: Operation cost for berth type $i$ in each year, which is supposed to be a percentage $l'_i$ of capital construction cost.

$l'' b_i^j$: Maintenance cost for gantry crane type $i$ in each year, which is supposed to be a percentage $l''_i$ of its price.

dredge: The annually cost of operation for dredging.

$$B = \sum_i \sum_m \sum_n \text{tariff}_{in} \times \beta_{in} \times x_{inn} \times f(P/F, r, n) \times f(P/A, r, t).$$  \hspace{1cm} (9)

$B$: Benefits earned by tariffs gained from foreign shipping lines.

tariff$_{in}$: Tariffs gained from ship type $i$ in year $n$.

$\beta_{in}$: The percentage of foreign ships type $i$ berthed in port in year $n$.

There are some other factors that must be calculated in port investment from national prospective such as environmental costs and benefits, job opportunities made by port development in the region, other infrastructural costs (roads, warehouses, reach stackers, etc.). In this study, though the goal is to find the optimum development so only the items related to the ship’s types will be considered. For instance, the number of needed warehouses strongly depends on the cargo forecast not on the ship type so they are not calculated or the job opportunities are indirectly related to the ship type so these benefits and costs were eliminated.

3.5. Constraints

Six constraints are considered in the model. Port operation constraints, integer programming constraints, dredging constraints, port capacity constraints, Shahid Rajaee’s world maritime container trade constraints and large ship constrains.

3.5.1. Port operation constraint

The amount of cargo imported to the port must be greater than the forecasted amount that must be handled.

$$\sum_i \sum_n x_{inn} \times T_i^j \geq d_n \hspace{1cm} \forall m = \{1, \ldots, m\} \hspace{0.5cm} \forall i = \{1, \ldots, I\} \hspace{0.5cm} \forall n = \{1, \ldots, N\}. \hspace{1cm} (10)$$
$T_i$: Total TEU that ship type $i$ can be loaded and unloaded.

$T_i = \text{load}_i + \text{unload}_i$.

$	ilde{d}_n$: Amount of cargo forecasted for year $n$. $\tilde{d}_n$ is a triangular fuzzy number and it includes total number of TEU, which must be loaded and unloaded in the port.

3.5.2. Port capacity constraints

\begin{equation}
R_i x_m \leq z_{in} u_t \times \text{day},
\end{equation}

\begin{equation}
R_i x_m + R_{i-1} x_{(i-1)n} \leq (z_m u_t + z_{(i-1)n} u_{i-1}) \times \text{day},
\end{equation}

\vdots

\begin{equation}
R_i x_m + R_2 x_{2n} + \cdots + R_1 x_{n} \leq (z_{in} u_t + \cdots + z_{2n} u_2 + z_{1n} u_1) \times \text{day}.
\end{equation}

$R_i$: Days needed ship’s type $i$ to berth, load, unload and separate. The amount of $R_i$ will be calculated by number of boxes to be loaded to or unloaded from each ship, number of gantries on the berth working on ship type $i$ and their efficiency, daily working hours of the port and hours needed to fasten and separate ships.

$z_{in}$: An integer variable used to model the number of berths needed for ship type $i$ in year $n$. The $q_{ik}$ is the difference between $z_{in}$ in the beginning and finishing year of interval $k$. For instance: $q_{i1} = z_{in1} - z_{in0}$.

$u_t$: Berth’s type $i$ occupancy factor.

day: The working days of the port during a year.

3.5.3. Dredging constraints

I. \[ b_i m y_{in} \leq \text{dredge}_n. \] (12)

$b_i m$: Initial dredging cost paid to deepen sea to a suitable depth for the ship’s type $i$. Therefore, the current depth of sea will be measured then the required draft and investment for candidate ships will be determined.\text{dredge}_n: A positive variable used to calculate dredging cost, which must be paid in year $n$. This constraint represents the maximum required dredging cost in year $n$ is calculated by the greatest cost needed for dredging.

II. \[ \text{dredge}_n' = \text{dredge}_n - \text{dredge}_n-1. \] (13)

\text{dredge}_n': A positive variable is used to calculate capital-dredging cost in year $n$ if the optimum berthing ships in that year needs greater draft than ships that were berthed in the port before.

3.5.4. World container trade of Shahid Rajaee port

\begin{equation}
\sum_i x_{imm} \times T_i \geq p_{mn} \tilde{d}_n.
\end{equation}

$p_{mn}$: Percentage of cargo in year $n$ that must be handled to or from Shahid Rajaee port by route $m$.

3.5.5. Large ships constraints

The building cost of mega ships is high so the number of mega ships is limited. This fact makes shipping lines to use mega ships carefully and only on some special routes. For this reason, the number of mega ships that will travel to this port is limited.

\begin{equation}
x_m \leq \eta_{im} \times \sum_i \sum_m x_{imm}, \quad i = 4, 5.
\end{equation}

$\eta_{im}$: Denotes the percentage of ships type $i$ in year $n$, which is possible to berth in our case study port.
3.5.6. Integer programming constraints

\[
\begin{align*}
  y_{in} & \leq x_{in}, \\
  x_{in} & \leq My_{in}, \\
  y_{in} & = \begin{cases} 
  1 & x_{in} > 0, \\
  0 & x_{in} = 0,
\end{cases}
\end{align*}
\]

where

\[
x_{in} = \sum_{m} x_{imn}.
\]

\(y_{in}\): 0–1 integer variable used to model the either–or situation.

\(m\): A large positive number.

In this study this constraint is used for the ships with capacity over 8000 TEU.

Taking into consideration, the maritime routes ending to or starting from Shahid Rajaee port have no dimensional limitation for our candidate ships. The only canal in their route is the Suez Canal, which the largest candidate ship (12,000 TEU) can pass.

4. Results

Julien’s approach is used to solve this fuzzy problem. In Table 2, \(\alpha\)-cuts of the cargo forecast study results in Shahid Rajaee port are given.

The model is developed using GAMS language and 10 cuts for \(\alpha\) from 0 to 1 by 0.1 steps are used. For each \(\alpha\)-cut the model presents the best way for development of Shahid Rajaee port. The results are given in four amounts of 0, 0.4, 0.8, 1.

4.1. \(\alpha = 0\)

For \(\alpha = 0\), the upper bound of optimum port development follows in Table 3. The net present value of total costs equals 5.15E + 9$ while the sum of investment cost (berth construction, gantry, dredging) and operation equals 691.261E + 6$. At the end of the study period (2030), 15 berths which are suitable for ships with capacity of 6000 TEU, five berths which are suitable for ships with capacity of 8000 TEU, four berths which are suitable for ships with capacity of 10,000 TEU and five berths which are suitable for ships with capacity of 12,000 TEU are needed. The length of berths constructed by government will be 11,570 m.

For \(\alpha = 0\), the lower bound of optimum port development follows in Table 4. According to Table 4 at the state of \(\alpha = 0\) the optimum investment is to construct berth type 5 in appropriate periods as Table 4. The net present value of total costs equals 2.29E + 9$.

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>2010</th>
<th>2015</th>
<th>2020</th>
<th>2025</th>
<th>2030</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>(d_L^a)</td>
<td>(d_U^a)</td>
<td>(d_L^a)</td>
<td>(d_U^a)</td>
<td>(d_L^a)</td>
</tr>
<tr>
<td>0</td>
<td>1445</td>
<td>2100</td>
<td>2200</td>
<td>4150</td>
<td>3000</td>
</tr>
<tr>
<td>0.1</td>
<td>1485.5</td>
<td>2075</td>
<td>2314</td>
<td>4096</td>
<td>3240</td>
</tr>
<tr>
<td>0.2</td>
<td>1526</td>
<td>2050</td>
<td>2428</td>
<td>3988</td>
<td>3480</td>
</tr>
<tr>
<td>0.3</td>
<td>1566.5</td>
<td>2025</td>
<td>2542</td>
<td>3907</td>
<td>3720</td>
</tr>
<tr>
<td>0.4</td>
<td>1607</td>
<td>2000</td>
<td>2656</td>
<td>3826</td>
<td>3960</td>
</tr>
<tr>
<td>0.5</td>
<td>1647.5</td>
<td>1975</td>
<td>2770</td>
<td>3745</td>
<td>4200</td>
</tr>
<tr>
<td>0.6</td>
<td>1688</td>
<td>1950</td>
<td>2884</td>
<td>3664</td>
<td>4440</td>
</tr>
<tr>
<td>0.7</td>
<td>1728.5</td>
<td>1925</td>
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<td>3583</td>
<td>4680</td>
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<tr>
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<tr>
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<td>1875</td>
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<td>3421</td>
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<tr>
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<td>1850</td>
<td>1850</td>
<td>3340</td>
<td>3340</td>
<td>5400</td>
</tr>
</tbody>
</table>
4.2. $a = 0.4$

For $a = 0.4$, the upper bound and the lower bound of optimum port development follows in Tables 5 and 6. The objective function varies in the range of $[2.85, 4.65]$ billion dollars. By increasing the amount of $a$, the interval of the net present value of total costs becomes narrower.

4.3. $a = 0.8$

For $a = 0.8$, the upper bound of optimum port development follows in Table 7 and the lower bound for optimum development will be as Table 8. When $a = 0.8$, then the net present value of costs from national view point will be in $[3.46, 4.08]$ billion dollars.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>The berths to be built in each sub period ($d = a_U^{d = 0}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
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<tr>
<td>2</td>
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<table>
<thead>
<tr>
<th>Table 4</th>
<th>The berths to be built in each sub period ($d = a_L^{d = 0}$)</th>
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<tbody>
<tr>
<td>5</td>
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</table>

<table>
<thead>
<tr>
<th>Table 5</th>
<th>The berths to be built in each sub period ($d = a_U^{d = 0.4}$)</th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
</tr>
<tr>
<td>5</td>
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<table>
<thead>
<tr>
<th>Table 6</th>
<th>The berths to be built in each sub period ($d = a_L^{d = 0.4}$)</th>
</tr>
</thead>
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<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
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<table>
<thead>
<tr>
<th>Table 7</th>
<th>The berths to be built in each sub period ($d = a_U^{d = 0.8}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
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<td>5</td>
<td>1</td>
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</table>
4.4. $x = 1$

For the case that $x = 1$, because the fuzzy number of cargo handling is triangular so that the lower and upper of the objective function will be the same. The optimum development when $x = 1$ will be as Table 9. Government has to construct 7260 m berth by the end of 2030. The national cost is 3.84 billion dollars (see Table 10).

4.5. Global results

The results of the model for each $x$ are given in Table 11.

By studying the results of the model shown in Table 11, some variations are observed. For instance, comparing the lower bound of berth lengths which must be constructed and investment costs for $x = 0.5$ and $x = 0.6$, in spite of the fact that berth lengths is increasing about from 4680 to 5640 m the construction cost decreases from 0.461E + 9$ to 0.428E + 9$, (about 33 million). As it was said $P$ is the amount of the net present value of investment so that the period of construction of different berths is important. The optimum construction schedule for $a = 0.5$, $d = a = 0.6$ which are yielded by solving the problem are given in Tables 12 and 13.

According to the devaluation rate (0.15), the time of investment is so important. For $x = 0.6$ the congestion of berth construction in last 10 years is higher but for $x = 0.5$ the opposite is correct. In fact, the higher investment in the future becomes less for the present.

The other point in Table 11 is that, for some cuts the optimum berth lengths goes down while the demand rises. For example for the lower bound of $x = 0.1$ and $x = 0.2$, the demand increases from 5,700,000 TEU to 6,400,000 TEU by the year 2030, while the optimum determined berth lengths goes down from 3640 m to 3430 m. The port capacity is related to the number of berths and the type of them. For $x = 0.1$ the optimum development is to construct 9 berths, using one berth type 1, three berth type 2 and five berth type 5 but for $x = 0.2$ the optimum development is to construct one berth type 1, one berth type 3, one berth type 4, five berth type 5. For $x = 0.2$ the number of berths with higher capacity is more than small berths. In Fig. 2 the fuzzy graph of objective function is illustrated.
In this research by using operational research science a fuzzy optimization model was formed where its objective function is to minimize the net present value of the investment on ports from national viewpoint.

Table 11
The output of the model
\[
\begin{array}{cccccc}
\alpha & G^L & G^U & P^L & P^U & BL^L & BL^U \\
0 & 2.29 & 5.15 & 0.299 & 0.687 & 2600 & 11,570 \\
0.1 & 2.45 & 5.01 & 0.320 & 0.668 & 3640 & 11,390 \\
0.2 & 2.55 & 4.92 & 0.336 & 0.702 & 3430 & 10,580 \\
0.3 & 2.71 & 4.79 & 0.369 & 0.697 & 3990 & 10,650 \\
0.4 & 2.85 & 4.65 & 0.361 & 0.665 & 4196 & 10,900 \\
0.5 & 3.04 & 4.52 & 0.461 & 0.618 & 4680 & 9530 \\
0.6 & 3.16 & 4.37 & 0.428 & 0.584 & 5640 & 8690 \\
0.7 & 3.29 & 4.24 & 0.433 & 0.579 & 5590 & 8476 \\
0.8 & 3.46 & 4.08 & 0.457 & 0.552 & 6420 & 8150 \\
0.9 & 3.66 & 3.9 & 0.432 & 0.512 & 6728 & 7420 \\
1 & 3.84 & 3.84 & 0.558 & 0.558 & 7260 & 7260 \\
\end{array}
\]

\(G^L\): The lower bound of objective function, net present value of costs from national view point (unit: 10^9 dollars).

\(G^U\): The upper bound of objective function, net present value of costs from national view point (unit: 10^9 dollars).

\(P^L\): The net present value of lower bound of investments (unit: 10^9 dollars).

\(P^U\): The net present value of upper bound of investment (unit: 10^9 dollars).

\(BL^L\): Lower bound of berth length (unit: m).

\(BL^U\): Upper bound of berth length (unit: m).

Table 12
The berths to be built in each sub period (\(d = d_\alpha^{0.6}\))

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Table 13
The berths to be built in each sub period (\(d = d_\alpha^{0.5}\))

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<td>5</td>
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</tbody>
</table>

Fig. 2. The variation of net present value of total costs by different \(\alpha\)-cut.

5. Conclusion

In this research by using operational research science a fuzzy optimization model was formed where its objective function is to minimize the net present value of the investment on ports from national viewpoint.
Analyzing port development investment as a fuzzy operational research problem is a new approach, applied in this research. Never before, there have been such an application of OR for port development and this is completely an innovative way for port development optimization. According to the uncertainties influencing the cargo forecast, fuzzy numbers are used to model the problem. Fuzzy logic application helps investor to know the value of membership degree of his development plan in the optimum set. In fact, for specific uncertainty, the port planner can understand the future optimum required berths, the optimum sub period of construction steps and his situation in the optimum decision. According to the results of this research, investors will be able to decide how to develop the port. For example, according their budget limitation or port development plan, they can decide on a specific membership degree ($\alpha$) during the first phase and develop the port based on lower or upper bound of that membership. Based on results of the model in first phase and comparing them with the actual level of port operation, for next phases they can use a different $\alpha$. The actual level of port operation may tend to the upper or the lower bound of a different $\alpha$. In comparison with classical models this model helps them to make a better decision in the next phases considering the suitable $\alpha$. As an example, investor has a budget limitation of 460E6$ for investment and 3.5E9$ for total costs from national viewpoint. According to the results of Table 11, the investor cannot choose $\alpha$ greater than 0.8. they will consider the results of Table 8 (lower bound) and for first 5 years, they must construct 1 berth type3 and purchase its needed facilities. At the end of 2010 they will survey the operation of the port, if there is no problem they will continue otherwise they must change their $\alpha$. On the other hand the owner of the port may want to develop port without considering the budget limitation. In this case they will act based on right side of Fig. 2.

Acknowledgement

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References

Lloyd’s register, 2000. Ultra large container ships, Lloyds, United Kingdom.