Time-Dependent Transportation Network Design Considering Land-Use and Equity Issue Between Land Owners With Ant Colony Optimization Method

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Abstract
Transportation network design is traditionally known as the problem of selecting the optimal capacity of links in a transportation network.
So far most transportation network design studies focus on the problem of optimizing the network for a certain time in future without considering time dimension, whereas huge economic growth and activities in transportation extent worldwide and rise in value of time infer to consider time dimension in studies. Time-dependant transportation network design is could be defined  as the gradual improvements to the network by considering the changing travel demand over a planning horizon. Even though network improvements could be beneficial for every network user, but it resultant change in land-use patterns and as a result change in land value may not be fare and even could be impairing for some land owners.
In this paper gradual network improvement and change in land-uses are considered simultaneously, and tried to optimize this problem in order to gain an acceptable level of inter land owners equity during the planning period with ACO method.
Keywords: Transportation network design, Time dependant, Land value, Equity, Ant colony optimization.

1. INTRODUCTION
The tremendous growth in traffic demand, which has been started since the 1920s, has caused major traffic problem and congestions such that all the effort of traffic engineers in terms of road construction couldn’t cope with this ever increasing travel demand. This inability, together with the limited availability of rescuers; specially budget, has led to the important of optimal decision on capacity improvement and enhancement projects, especially in developing countries.
Due to this need the Network Design Problem (NDP), which is the problem of selecting the optimal capacities in transportation network, has gained attention science mid 70s. NDP has could be classified into three broad categories; Continuous Network Design Problem (CNDP), Discrete Network Design Problem and Mix Network Design Problem. In the CNDP the decision variables, link capacities, are considered to be continuous variables. In the DNDP the decision is on whether or not to implement a predetermined capacity enhancement project. MNDP is a NDP that contains both discrete and continuous variables. Theoretically the DNDP could be considered as a special case of CNDP. Although more realistic, DNDP has been less considered in previous studies. Boyce and Janson (1980) [1] noted that the result obtained by the solution of CNDP or somehow unrealistic. LeBlanc and Abdullah (1979) [2] have also reported more flexibility in the solution of DNDP. However the DNDP spheres for the problem of very high run times that has made it impractical for large scale applications [2].
NDP has been traditionally formulated as a bi-level optimization problem. The upper level problem, that is usually called the design problem or the leader problem, deals with the designer’s objective which is usually to minimize some performance measure of the network such as total travel time. The lower level problem, the follower or the user problem, models the reaction of users to the changes imposed to the network by the designer. This problem is usually considered as a User Equilibrium (UE) Traffic assignment problem. This structure of the NDP makes the resultant problem non-convex and non-smooth which adds to the intractability of its solution.
In terms of solution methods used for the NDP, Meng et al. (2001)[3] solved the NDP by the
introduction of a merit function and have converted the bi-level optimization problem into a single level one.
Lo and Szeto (2009) [4] have considered the time dimension into the NDP, this variation of NDP is the major
focus of this paper. Chen and Alfa (1991) [5] and Davis (1994) [6] used stochastic user equilibrium in the
NDP. Heydacker (2002) [7] used dynamic user equilibrium traffic assignment problem for network design,
which only seem to add to the complexity of the problem. Lo (2002) [8] and Lo & Tang (2003) [9] have used a
so called probabilistic user equilibrium problem for the solution of NDP. Boyce and Janson (1980) [1] Lam
(2002) [10] used a combined trip assignment and distribution method to solve the NDP.

From the objective function point of view, Maher et al. (2001) [11] used the total travel time as their
objective function for NDP. Miandoabchi and Farahani (2011) [12] used the maximization of the reserve
capacity of the network. Lam (2002) [10] and Szeto (2005) [13] have considered the maximization of
consumer surplus as their objective function. Multi objective programming has been also considered in some
previous studies such as Friesz and Harker (1983) [14], Current and Min (1986) [15], Tzeng and Tsaur
(1997) [16] and Szeto (2008) et al. [17].

Most road and transportation construction project are large scale in nature and thus, the order in which
this projects should be implemented and the availability of budget in specific time intervals are important
aspect in this decision process. Moreover, the long term effects of transportation decisions (e.g. land use and
effect) necessitate the consideration of a time dimension in NDP. As mentioned previously the time
dimension hasn’t been considered in most previous researches. Only Szeto has considered the effect of time
on the network design as NDP-T in terms of budget sensitivity [13], reliability [4], equity [18], land use
effects [19], tolling strategies [17] and cost recovery [4].

In this paper NDP-T model has been introduced. This model in spite the ones used by Szeto is a bi-
level optimization model and considers the equity of land owner in term of equitable accessibility
improvements. This paper uses Ant Colony Optimization algorithm (ACO) to solve the NP-hard NDP-T.
Szeto has used Generalized Reduced Gradient (GRG) in his previous attempts to solve the NDP-T that is
time consuming and impractical for large scale applications. Mean while due to the non-convex nature of the
problem the GRG only gives local optimal solutions to the problem while one can hope to find a better
solution by the use of ACO.

2. MODEL FORMULATION

In this paper the NDP-T has been modeled as a bi-level optimization problem. Which as mentioned
previously the upper level is the design problem and the lower level problem is the UE traffic assignment
problem. This model could be expressed, in general as in equation (1):

\[
(ULP) \quad \text{Max } F(X, f(u, Q)) = \begin{cases} 
F_1(X, f(u, Q)) \\
F_2(X, f(u, Q)) \\
\vdots \\
F_n(X, f(u, Q))
\end{cases}
\]

\[
\text{s.t. } H(X, f(u, Q)) \leq 0
\]

\[
(LLP) \quad \text{Min } f(u, Q)
\]

\[
\text{s.t. } h(u, Q) \leq 0
\]

Where ULP is the upper level problem and the lower level problem in denoted by LLP. In this general
formulation the NDP-T has been formulated as a multi objective optimization problem, which the ULP
objective function vector \( F \) consist of several other objectives namely, \( F_1, F_2, F_3, \ldots, F_n \) Subject to the
constraint vector \( H \), the lower level problem is the elastic demand UE traffic assignment problem that is
subjected to conservation of flow constraints (h). In this problem \( X \) is the vector of design variables, \( u \) is the
vector of path flows and \( Q \) is the demand vector in the network.

Based on this general formulation the NDP-T in this paper has been formulated as follows. Given a directed
network graph such as \( G(N,A) \), \( N \) is the set of all nodes and \( A \) is the set of all links in the network. Then the
Elastic demand user equilibrium model has been formulated as follows, as the LLP:

\[
\text{Min } \sum_a f_{a}^{r}(w)D^{-1}_{a}D_{a}\sum_{OD}t_{a}(w)d\text{w}
\]

\[
s.t.: u_{p}^{OD} = \sum_{a}t_{a}^{OD}\delta_{a,p}^{OD}
\]

\[
V_{a} = \sum_{OD}f_{p}^{OD}\delta_{a,p}^{OD}
\]
\[
\sum_{p} f_{pt}^{OD} = q_{t}^{OD} \tag{5}
\]
\[
f_{pt}^{OD} \geq 0 \tag{6}
\]
\[
q_{t}^{OD} \geq 0 \tag{7}
\]
\[
D_{OD} = q_{t}^{OD} G_{OD} A_{p} g_{OD} (u_{t}^{*OD}) \tag{8}
\]

This problem is the user equilibrium with elastic demand, \(D_{OD}^{-1}(\cdot)\) is the inverse demand function. Equation (3) indicates that the travel time of each path \(p\) is the sum of travel time of the links that are contained in it. \(u_{pt}^{OD}\) is the travel time of path \(p\) between the OD pair and \(t_{a}\) is the travel time on link \(a\). \(\delta_{ap}^{OD}\) is a binary variable which takes the value one if link \(a\) belongs to path \(p\) and zero other wise. Equation (4) states that the flow on each link is the sum of the flows on the all paths passing though the link. Equation (5) is the conservation of flow constraint. Equation (8) is the demand function, \(D_{OD}\), which is a function of shortest path travel time, \(g_{OD} (u_{t}^{*OD})\), and the amount of trip attraction and production.

The ULP objective is to maximize the total system revenue in terms of total benefits attained by the construction of the new projects (e.g. saving in travel time) minus the total construction cost as in equation (9):
\[
\text{Max } Z = TIB - TIC \tag{9}
\]

In this equation \(TIB\) is the total benefit and \(TIC\) indicates the total construction cost. The ULP is subjected to the following constraints.
\[
TIC \leq TB + TT \tag{10}
\]
\[
TIC = \sum_{t} IC_{t} \tag{11}
\]
\[
IC_{t} = \sum_{a} NL_{a} L_{a} \alpha_{1} \tag{12}
\]
\[
L_{a} = t_{a0} \alpha_{2} \tag{13}
\]
\[
NL_{a} \leq NL_{a, \text{max}} \tag{14}
\]
\[
\sum_{a} NL_{a} \leq IP_{t} \tag{15}
\]
\[
IC_{t} \leq TB_{t} \tag{16}
\]
\[
TB_{t} = TB - \sum_{1}^{t-1} (IC_{t} + TT_{l}) \tag{17}
\]
\[
TIB = \phi(BNTT - INTT) \alpha_{3} \tag{18}
\]
\[
BNTT = \sum_{i} \sum_{a} V_{at} t_{a} \tag{19}
\]
\[
INTT = \sum_{i} \sum_{a} V_{at} \bar{t}_{a} \tag{20}
\]

The first constraint indicates that the total construction cost should be less than or equal to the total budget, \(TB\), plus the revenue gained for the road tolls or from congestion charging, \(TT\). In equation (11), \(IC_{t}\) is the total construction cost in time period \(t\). The total construction cost in each time period, as in equation (19), is equal to the number of lanes constructed, \(NL_{a}\), times the total length of the section, \(L_{a}\), times unit cost of construction, \(\alpha_{1}\), which has been assumed to be equal for all road segments. Equation (13) shows total length of each section as a function of section’s free flow travel time. The number of lanes constructed on each road is constraint to an upper limit that indicated technological, topographical, and practical or safety limitation. \(IP_{t}\) indicates the total number of lanes that could be constructed in each time period. Obviously the total cost in each time period, \(IC_{t}\), should less than or equal to the total budget allocated to that time period, \(TB_{t}\), which has been given in equation (16). The budget allocated to each time period, as given in equation (17), depends on the total budget, the amount of budget expended in the previous time periods and total revenue gained by the tolls in the previous time periods, \(TT_{l}\).
All other equation given in the above problem is definition function. In equation (18) $TIB$ is given as a function of total travel time in the base year, $BN_{TT}$, and the total time period, $INTT$, $\phi$ is the value of time and $\alpha_3$ is a parameter that converts hourly travel cost to annual travel time.

In this paper equity has been considered as the total per capita benefit derived from the road network improvement, $\bar{NB}_i$, in each zone, $i$. As a constraint, it has been assumed that the per capita improvement, $\Delta NB_i$, should be equal by a very small allowance, between all zones in the network, this has been given in equation (21). In equations (23) population in each year has been raised by a growth factor, $PG_i$. The total benefit in the network, $NB_i$, is difference between the weighted sum of shortest path travel times in the base condition and final network, as given in equations (24),(25) and (26).

$$\Delta NB_i = \|NB_i - NB_j\| \leq \varepsilon \quad \forall i, j \in N \ i \neq j$$

$$\bar{NB}_i = \frac{\sum n_{it}}{N}$$

$$n_{it} = PG_i n_{it-1}$$

$$NB_i = BNC_i - INC_i$$

$$BNC_i = \phi \left( \sum_{i} \sum_{D} \sum_{P} f_{OD}^{pt} u_{pt}^{OD} \right)$$

$$INC_i = \phi \left( \sum_{i} \sum_{D} \sum_{P} f_{OD}^{pt} u_{pt}^{OD} \right)$$

The presented model has been solve using ACO, the details of application of ACO to the proposed model has been omitted from this paper for sake of abbreviation. Interested readers are referred the MS thesis of the second author for more information on the details of ACO or the some more specialized text in this field [20].

3. TEST NETWORK

The test network used in this study has been shown in Fig (1). In this network the two candidate links are 8 and 9 and both links could be candidate for toll collection. The origin destination matrix has been given in Tab (1). The design period has been assumed to consist of five, two year periods (which sum ups to 10 years). The growth factors associated to the number of trips and populations are as follows:

$G_1 = G_3 = 1.11 \quad & \quad G_2 = 1.08 \quad & \quad A_1 = 1.03 \quad & \quad A_2 = 1.06$

$PG_1 = 1.016 \quad & \quad PG_2 = PG_3 = 1.014 \quad & \quad n_1 = n_2 = 5000 \quad & \quad n_3 = 3000$

The demand function is given in equation (26). The free flow travel time and the initial capacity of the links in the network as given in Tab(2). All other parameters needed for model implementation are given in tab (3).

$$f_{OD} = 1.02 (u_{i}^{OD})^{0.03}$$

Figure 1. Test Network

Table 1- OD Matrix

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1</td>
<td>1500</td>
<td></td>
</tr>
<tr>
<td>O2</td>
<td>1000</td>
<td>1500</td>
</tr>
<tr>
<td>O3</td>
<td></td>
<td>2000</td>
</tr>
</tbody>
</table>


has set such that the benefit experienced by each origin would be quite similar. This constraint guarantees the resultant capacities and flows obtained from the application of the unconstraint model are shown in Table (6) and (7) respectively. The equity constraint guaranties that the capacity of links 4, 5 and 9 has been expanded in the first periods to have more benefit than in the subsequent time periods. An interesting point in the solution is that the capacity expansions in the first periods have shown to have more benefit than in the latter periods. The capacity of links 4, 5 and 9 has been adjusted to have more benefit in the first periods.

### Table 2- Test network parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB</td>
<td>185 × 10^9 Rials</td>
</tr>
<tr>
<td>α₁</td>
<td>0.5</td>
</tr>
<tr>
<td>α₂</td>
<td>0.33</td>
</tr>
<tr>
<td>α₃</td>
<td>2 × 24 × 365 = 17520</td>
</tr>
<tr>
<td>α₄</td>
<td>2000</td>
</tr>
<tr>
<td>α₅</td>
<td>1.0</td>
</tr>
<tr>
<td>α₆</td>
<td>0.15</td>
</tr>
<tr>
<td>α₇</td>
<td>4.0</td>
</tr>
<tr>
<td>φ</td>
<td>40 × 10^8 Rials</td>
</tr>
<tr>
<td>Tₘₐₓ</td>
<td>20 × 10^8 Rials</td>
</tr>
</tbody>
</table>

The model has been applied twice, once with the equity constraint and once without the equity constraint. The resultant capacities and flows obtained from the application of the unconstraint model are shown in Table (6) and (7) respectively. The first column in the base condition and the following columns are the flows on each link in the network.

### Table 4- Resultant capacities

<table>
<thead>
<tr>
<th>Link</th>
<th>Base Situation</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
<th>Period 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td>4000</td>
<td>4000</td>
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<td>2</td>
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<td>2000</td>
<td>2000</td>
<td>4000</td>
<td>4000</td>
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<tr>
<td>4</td>
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<td>3000</td>
<td>3000</td>
<td>3000</td>
<td>4000</td>
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<tr>
<td>5</td>
<td>3000</td>
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<td>6</td>
<td>3000</td>
<td>3000</td>
<td>4000</td>
<td>4000</td>
<td>4000</td>
<td>4000</td>
</tr>
<tr>
<td>7</td>
<td>3000</td>
<td>3000</td>
<td>4000</td>
<td>4000</td>
<td>4000</td>
<td>4000</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>3000</td>
<td>3000</td>
<td>6000</td>
<td>6000</td>
<td>6000</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>3000</td>
<td>3000</td>
<td>5000</td>
<td>5000</td>
<td>6000</td>
</tr>
</tbody>
</table>

### Table 5- Link flow

<table>
<thead>
<tr>
<th>Link</th>
<th>Base Situation</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
<th>Period 5</th>
</tr>
</thead>
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<td>2721</td>
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<td>3463</td>
<td>3771</td>
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<tr>
<td>2</td>
<td>1118</td>
<td>1864</td>
<td>2220</td>
<td>2410</td>
<td>2751</td>
<td>3109</td>
</tr>
<tr>
<td>3</td>
<td>1382</td>
<td>1997</td>
<td>2301</td>
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<td>3017</td>
<td>3784</td>
<td>4860</td>
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</table>

As could be seen the model has given more emphasis on capacity expansion in the first three periods. This is because capacity expansions in the first periods have shown to have more benefit than in the subsequent time periods. An interesting point in the solution is that the capacity of links 4, 5 and 9 has been altered in the fourth and fifth time periods. These improvements have been feasible due to the revenue gained from the tolls collected at the freeways. The accumulation of these tolls has compensated the construction cost of the new links.

In the second stage the model has been tested with the equity constraint. The capacities and flows obtained by the application of the model are shown in Table (6) and (7) respectively. The equity constraint has set such that the benefit experienced by each origin would be quite similar. This constraint guarantees equitable improvements in the network.
As could be expected the result of the two models are different. For instance although the capacity of the freeways could be exceeded up to 6000 vehicle per hour, but due to the equity constraints the capacity has been expanded to only 5000 vehicle per hour. Besides, the capacity expansion of the links shows a more gradual rate than the case without the equity constraint. Although the objective function value has apparently decreased as the result of the addition of equity constraint, but the results are more equitable. These differences are more obvious in the last time periods. This shows that the equity constraint has forced the model to invest in project that more advantages for some origins that the network as a whole.

### Table 6- Resultant capacities in equity solution

<table>
<thead>
<tr>
<th>Link</th>
<th>Base Situation</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
<th>Period 5</th>
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### Table 7- Link flow in equity solution

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<th>Period 3</th>
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<td>2665</td>
<td>3581</td>
<td>4441</td>
<td>4711</td>
</tr>
</tbody>
</table>

As could be expected the result of the two models are different. For instance although the capacity of the freeways could be exceeded up to 6000 vehicle per hour, but due to the equity constraints the capacity has been expanded to only 5000 vehicle per hour. Besides, the capacity expansion of the links shows a more gradual rate than the case without the equity constraint. Although the objective function value has apparently decreased as the result of the addition of equity constraint, but the results are more equitable. These differences are more obvious in the last time periods. This shows that the equity constraint has forced the model to invest in project that more advantages for some origins that the network as a whole.
Figure (2) shows the improvements gained by the application of the model in each time period. As could be seen the total benefit is lower for the equity constraint case in comparison to the unconstraint case. This is due to the additional equity constraint. Figures (3), (4) and (5) illustrate the differences in the benefits in each origin in comparison to the unconstraint case. As could be seen the differences are negligible as a result of the introduction of the equity constraints.

4. **CONCLUSIONS**

As the time dimension has been ignored in many past researches on network design, in this paper this dimension has been added to the traditional network design problem. The introduction of time in the network design requires major changes in the more familiar network design problem formulation and solution. These changes have been applied in the model presented in this paper by the use of new budget constraints and the use of variable demand formulation of the network design. Meanwhile the increased number of tips has been taken into account by the consideration of population growth factors which implies more trip in the network. An additional equity constraint has been also introduced in the presented model. The model has been then applied to a simple case study to illustrate its capabilities. The results have shown major improvements in the equity of the network and the total benefits derived from the network design. The use of other equity measures such as temporal equity is suggested for future researches. Meanwhile other optimization tools such as Genetic algorithm could be also used to solve the master problem given in this paper.

5. **REFERENCES**